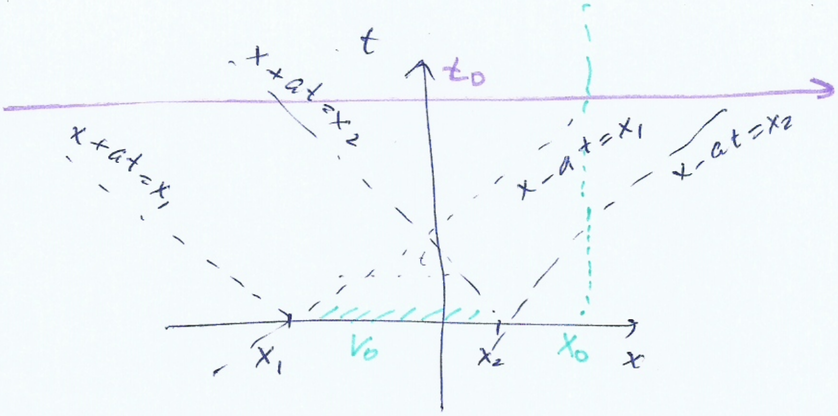
№2

$u_{tt} = a^2 u_{xx}$

$u(x,0) = 0$

$u_t(x,0) = \begin{cases} 0, & x < x_1 \\ V_0, & x_1 < x < x_2 \\ 0, & x_2 < x \end{cases}$



По ф-ле Даламбера: $u(x,t) = \frac{1}{2a}\int\limits_{x-at}^{x+at} u_t(\xi,0)\,d\xi$

$x = x_0 \quad x_0 > x_2$

$u(x_0,t) = \begin{cases} 0 & t < \frac{x_0-x_2}{a} \\ \frac{1}{2a}\int\limits_{x_0-at}^{x_2} V_0\,d\xi & \frac{x_0-x_2}{a} \le t \le \frac{x_0-x_1}{a} \\ \frac{1}{2a}\int\limits_{x_1}^{x_2} V_0\,d\xi & \frac{x_0-x_1}{a} \le t \end{cases} = \begin{cases} 0 & t < \frac{x_0-x_2}{a} \\ \frac{V_0}{2a}(x_2 - x_0 + at), & \frac{x_0-x_2}{a} \le t \le \frac{x_0-x_1}{a} \\ \frac{V_0}{2a}(x_2 - x_1), & t > \frac{x_0-x_1}{a} \end{cases}$

$t = t_0 \quad t_0 > \frac{x_2-x_1}{2a}$

$\begin{cases} x - at = x_1 \\ x + at = x_2 \end{cases}$

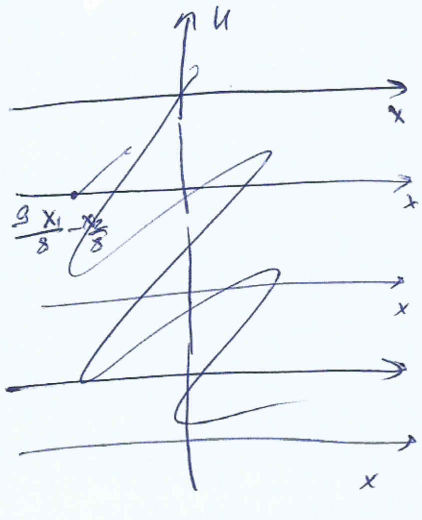
$at + at = x_2 - x_1$

$t = \frac{x_2-x_1}{2a}$ - точка пересечения прямых => мы выше

$u(x,t_0) = \begin{cases} 0, & x \le x_1 - at_0 \\ \frac{V_0}{2a}(x + at_0 - x_1), & x_1 - at < x \le x_2 - at \\ \frac{V_0}{2a}(x_2 - x_1), & x_2 - at < x \le x_1 + at \\ \frac{V_0}{2a}(x_2 - x + at_0), & x_1 + at < x \le x_2 + at \\ 0 & x > x_2 + at \end{cases}$

$t_k = \frac{x_2 - x_1}{8a} k \quad k = \overline{0,4}$

$x = x_1 + at_k$
$x = x_2 - at_k$



$k = 0$

$k = 1$

$k = 2$

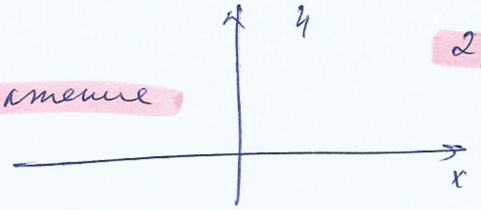
$k = 3$

$k = 4$

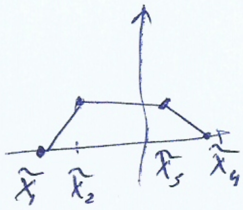
ДЗ 9

№2 продолжение

$k=0$



$k=1,2,3,4$ общ случ:



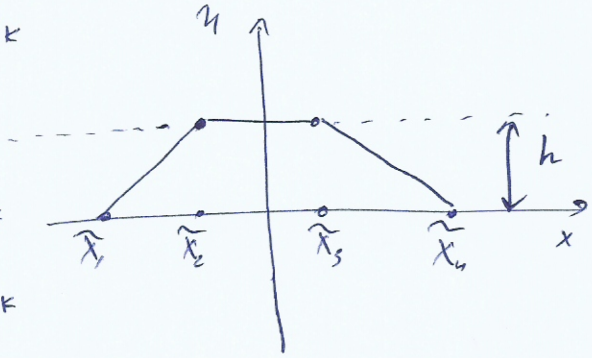
$k=1$

$$\tilde{x}_1 = x_1 - \frac{x_2-x_1}{8} = x_1 - at_k$$

$$\tilde{x}_2 = x_1 + \frac{x_2-x_1}{8} = x_1 + at_k$$

$$\tilde{x}_3 = x_2 - \frac{x_2-x_1}{8} = x_2 - at_k$$

$$\tilde{x}_4 = x_2 + \frac{x_2-x_1}{8} = x_2 + at_k$$



$$\frac{1}{2a}\int_{x_1}^{?} v_0\,d\xi = \frac{v_0}{2a}(? - x_1) = h$$

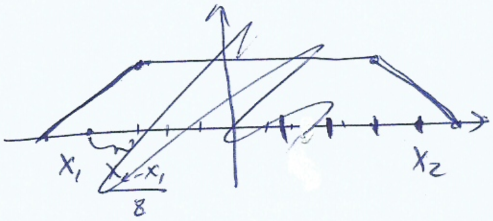
$? = x_0 + at_0 \qquad x_0 - at_0 = x_1$

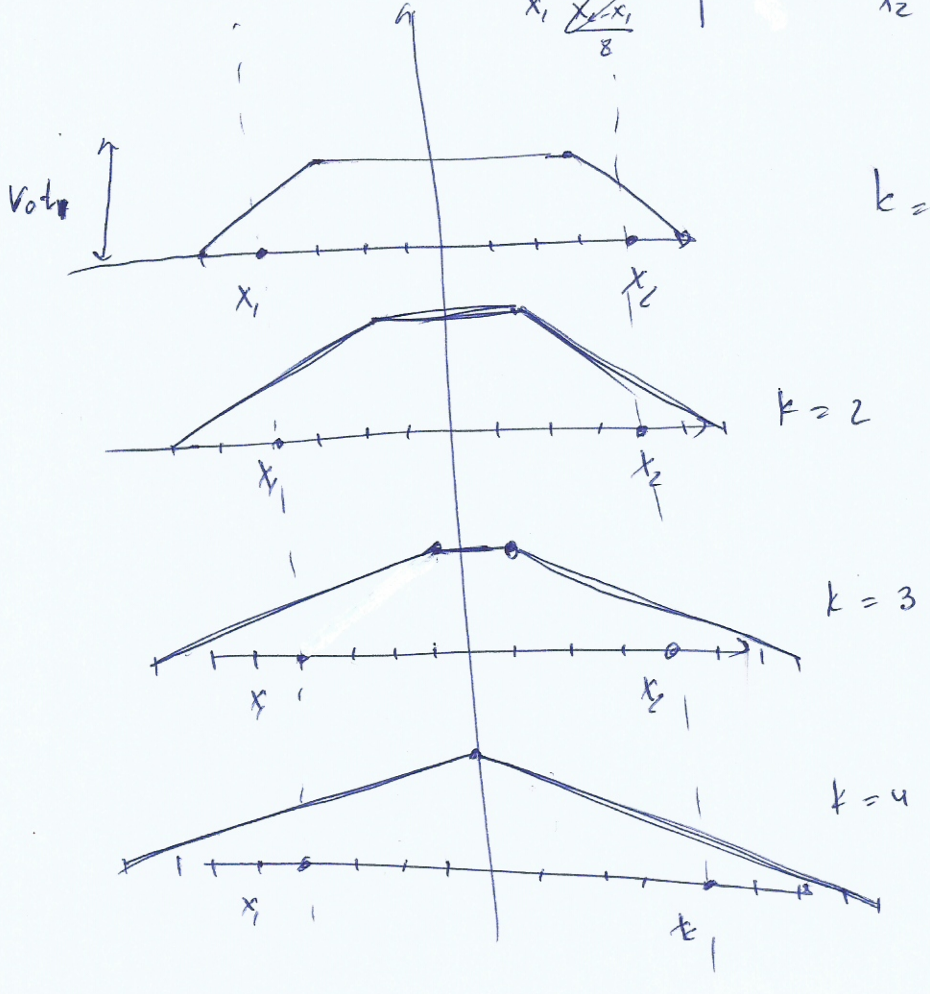
$x_0 = x_1 + at_0$

$$? = x_1 + at_0 \cdot 2 = x_1 + \frac{x_2-x_1}{4}$$

$$h = \frac{v_0}{2a}\left(\frac{x_2-x_1}{4} + x_1 - x_1\right) = \frac{v_0(x_2-x_1)}{8a} = v_0 t_k$$

$k=1$





ДЗ 9 3/4

№ 3

$u_{xx} - 4u_{xy} + 5u_{yy} - 3u_x + u_y + u = 0$

$A = 1, \quad B = \cancel{2} \quad C = 5$

$B^2 - AC = -1 < 0$ — эл. тип.

$dy = \frac{-2 \pm i}{1} \cdot dx$

$y = (-2 \pm i)x + C$

$\xi = y + 2x$

$\eta = x$

$$\begin{array}{r|l} 1 & u_{xx} = u_{\xi\xi} \cdot 2^2 + 2u_{\xi\eta} \cdot 2 + u_{\eta\eta} \cdot 1 \\ 5 & u_{yy} = u_{\xi\xi} \cdot 1 + 0 + 0 \\ -4 & u_{xy} = u_{\xi\xi} \cdot 2 + u_{\xi\eta}(\cdot 1) + u_{\eta\eta} \cdot 0 \\ -3 & u_x = u_\xi \cdot 2 + u_\eta \cdot 1 \\ 1 & u_y = u_\xi \cdot 1 + u_\eta \cdot 0 \end{array}$$

$u_{\xi\xi}(4 + 5 - 8) + u_{\xi\eta}(4 - 4) + u_{\eta\eta}(1) - 5u_\xi - 3u_\eta + u = 0$

$u_{\xi\xi} + u_{\eta\eta} - 5u_\xi - 3u_\eta + u = 0$

Чтобы изб. от мл.-их произв. $u(\xi, \eta) = v(\xi, \eta) e^{\alpha\xi + \beta}$

$v_{\xi\xi} e^{\alpha\xi + \beta\eta} + 2v_\xi \alpha e^{\alpha\xi + \beta\eta} + v e^{\alpha\xi + \beta\eta} \cdot \alpha^2 + v_{\eta\eta} \cdot e^{\alpha\xi + \beta\eta} + 2v_\eta \beta e^{\alpha\xi + \beta\eta} + v\beta^2 e^{\alpha\xi + \beta\eta}$

$- 5(v_\xi e^{\alpha\xi + \beta\eta} + v\alpha e^{\alpha\xi + \beta\eta}) - 3(v_\eta e^{\alpha\xi + \beta\eta} + v\beta e^{\alpha\xi + \beta\eta}) + ve^{\alpha\xi + \beta\eta} = 0$

$v_\xi: \ 2\alpha - 5 \qquad v_\eta: 2\beta - 3 \qquad v_{\xi\xi}: 1 \quad v_{\eta\eta}: 1 \quad v: \alpha^2 + \beta^2 - 5\alpha - 3\beta + 1$

$\alpha = 2{,}5 \quad \beta = 1{,}5 \qquad \left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2 - 5 \cdot \frac{5}{2} - 3 \cdot \frac{3}{2} + 1 = \frac{25 + 9}{4} - \frac{25 + 9}{2} + 1 =$

$= \frac{34}{4} - \frac{34}{2} + 1 = \frac{17}{2} - 17 + 1 = -16 + \frac{17}{2} =$

$= -16 + \frac{16}{2} + \frac{1}{2} = -8 + \frac{1}{2} = -\frac{15}{2}$

$$\boxed{v_{\xi\xi} + v_{\eta\eta} - \frac{15}{2} v = 0}$$

ДЗ 9

№4

$$\begin{cases} u_{tt} = a^2 u_{xx} + \beta x^2 & x \in \mathbb{R} \quad t > 0 \\ u(x,0) = e^{-x} & x \in \mathbb{R} \\ u_t(x,0) = \gamma & x \in \mathbb{R} \end{cases}$$

по ф-ле Даламбера

$$u(x,t) = \frac{1}{2}\left(e^{-(x-at)} + e^{-(x+at)}\right) + \frac{1}{2a}\int_{x-at}^{x+at} \gamma \, d\xi + \frac{1}{2a}\int_0^t d\tau \int_{x-a(t-\tau)}^{x+a(t-\tau)} \beta \xi^2 d\xi =$$

$$= \frac{e^{at-x} + e^{-at-x}}{2} + \gamma t + \frac{\beta}{6a}\int_0^t (x+a(t-\tau))^3 - (x-a(t-\tau))^3 \, d\tau =$$

$$= \frac{e^{at-x} + e^{-at-x}}{2} + \gamma t + \frac{\beta}{6a}\int_0^t 2a(t-\tau)\left((x+a(t-\tau))^2 + (x+a(t-\tau))(x-a(t-\tau)) + (x-a(t-\tau))^2\right) d\tau$$

$$= \frac{e^{at-x} + e^{-at-x}}{2} + \gamma t + \frac{\beta}{3}\int_0^t (t-\tau)\left(3x^2 + (a(t-\tau))^2\right) d\tau =$$

$$= \frac{e^{at-x} + e^{-at-x}}{2} + \gamma t + \frac{\beta}{3}\left[-\frac{3x^2(t-\tau)^2}{2}\Big|_0^t - \frac{a^2(t-\tau)^4}{4}\Big|_0^t\right] =$$

$$= \frac{e^{at-x} + e^{-at-x}}{2} + \gamma t + \frac{\beta x^2 t^2}{2} + \frac{\beta a^2 t^4}{12}$$

№5

$$\begin{cases} u_{tt} = a^2 u_{xx} & x \in \mathbb{R} \quad t > 0 \\ u(x,0) = u_0 \sin(kx) \\ u_t(x,0) = u_0 a k \cos(kx) \end{cases}$$

По ф-ле Даламбера

$$u(x,t) = \frac{1}{2}\left(u_0 \sin(k(x-at)) + \sin(k(x+at))u_0\right) + \frac{1}{2a}\int_{x-at}^{x+at} u_0 a k \cos(k\xi)\, d\xi =$$

$$= \frac{u_0}{2}\left[\sin(k(x-at)) + \sin(k(x+at)) + \sin(k(x+at)) - \sin(k(x-at))\right] =$$

$$= u_0 \sin(k(x+at))$$